MECHANICAL WOOD PROCESSING AND WOOD TECHNOLOGY

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V. K. Pashkov, S. V. Shchepochkin

Valentin Kuzmich Pashkov was born in 1933, graduated from Leningrad Forestry Academy in 1955, Doctor of Technical Sciences, Professor of Woodworking Machinery and Tools Department at Ural State Forest Engineering University. He has approximately 300 publications in the field of thermal physics of wood cutting.

Sergei Vladimirovich Shchepochkin was born in 1981, graduated from Ural State Forest Engineering University in 2003, Assistant Professor of Woodworking Machinery and Tools Department at Ural State Forest Engineering University. He has approximately 20 publications in the field of thermal physics of wood cutting.

TEMPERATURE FIELD OF CIRCULAR SAW TOOTH

Analytical calculations of temperature distribution along the height of circular saw tooth are performed. The character of distribution is shown to be similar to the one determined experimentally.

Keywords: circular saw, disc, tooth, temperature.

The development of analytical methods for the calculation and distribution of temperatures on the contact surfaces of saw tooth blade and along its height is an important task of thermal physics of wood cutting with circular saws. The solution to this problem allows to develop recommendations related to the wear and durability of the cutting edges and surfaces of the blade, thermal stresses in the tool, determination of cutting modes for which it is advisable to use tooling materials with varying degrees of temperature stability.

Analytical studies cannot be performed without certain schematization of heat transfer during the movement of chips along the tooth blade and the blade along the cutting surface [4].

In the cylindrical coordinate system *r, φ, z* that is associated with the heat source, the differential equation of heat conduction in the general case is as follows:

$$\left[\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\frac{\partial t}{\partial r}+\frac{1}{r^2}\frac{\partial^2 t}{\partial \varphi^2}+\frac{\partial^2 t}{\partial z^2}\right]+\left[\left(\frac{\partial t}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial t}{\partial \varphi}\right)^2+\left(\frac{\partial t}{\partial z}\right)^2\right]\frac{\partial \lambda}{\partial t}=$$
$$=C_V\left[\frac{\partial t}{\partial \tau}+V_r\frac{\partial t}{\partial r}+V_\varphi\frac{\partial t}{\partial \varphi}+V_z\frac{\partial t}{\partial z}\right], \quad (1)$$

where $t(r, \varphi, z, \tau)$ – is the temperature of the body under consideration at the point with coordinates r, φ, z at the moment of time τ;
C_V – is the volumetric heat capacity;
V_r, V_φ, V_z – are the radial, tangential and axial components of velocity vector of the body relatively to the heat source;
$X(t)$ – is the coefficient of thermal conductivity of the material of the body.

The solution of the thermal conductivity equation (1), as well as any other non-linear partial differential equation, is associated with significant difficulties. Therefore, the most important step in thermophysical analysis is the simplification of the structure of equation (1) based on the schematization of the process under consideration.

With regard to the heating of the cutting part of the saw, which is rotating around an axis z, when V_r, V_z = 0, it is reasonable and, according to the available data, permissible to consider that its temperature field is flat $\frac{\partial t}{\partial z}; \frac{\partial^2 t}{\partial z^2}=0$ (due to the small thickness of the disk), axisymmetric $\frac{\partial t}{\partial \varphi}; \frac{\partial^2 t}{\partial \varphi^2}=0$ (in the range of cutting speeds applied in woodworking [1, 5]) and stationary $\frac{\partial t}{\partial \tau}=0$ (with interfacial gaps not exceeding 10% of the length of work pieces to be cut [6]). In addition, with the accepted stationary temperature field, the influence of disk temperature on the thermophysical properties of its material can be neglected [4].

The loss of heat by the tooth blades as a result of irradiation does not exceed, according to [2], 1.0—1.5% of the total amount of heat entering the tool and participating in heat exchange processes with the environment.

Taking into account the above limitations and heat transfer processes along the side surfaces and surfaces of the front and rear faces of the saw blade, let us consider the heat balance of an elementary annular layer with an inner radius x and a centre at the point *n* of the cutting edge nn_1 of absolutely sharp blade with a width dx (see Fig. 1). A profile of an absolutely sharp blade is superimposed on the profile of a physical blade with a radius of curvature ρ and a cutting edge in the shape of an arc *аб* with a centre O.

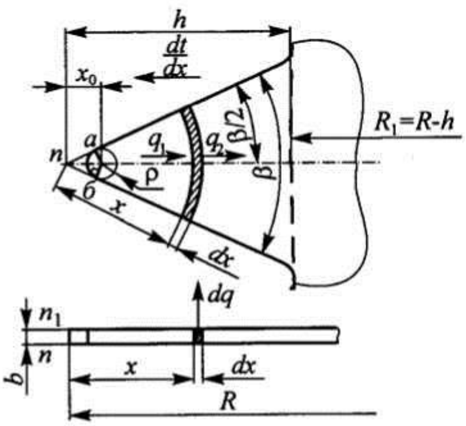


Figure 1. Scheme of heat transfer flows of an elementary segment of circular saw tooth

Below are the designations used in Fig. 1: $\frac{dt}{dx}$ – is the temperature gradient along the height of circular saw tooth; q_1 and q_2 – is the amount of input and output heat from the elementary segment due to thermal conductivity along the height of the tooth, W; *dq* – the amount of heat removed by convective thermal exchange from an elementary segment through the front, rear and side faces, W; ß – is the tooth wedge angle, degrees; *h* and *b* – are the height and thickness of the tooth, m; *R* and R_1 – are the radius of the saw and the dedendum circle, m; x_0 – is the distance from the origin of coordinates (point *n)* to the centre of the arc *аб* (point O).

Since in the steady-state cutting mode the heat content of the elementary segment remains constant, the heat transfer flow *dq*, which is dissipated by means of convection by side surfaces and surfaces of the front and rear faces of this section, is determined by the difference in heat transfer flows q_1 and q_2 passing through the outer and inner cylindrical surfaces:

$$dq = q_1 - q_2. \tag{2}$$

The members of the right side of expression (2) are as follows:

$$q_1 = -2\pi x b \frac{\beta}{360} \lambda \frac{dt}{dx}; \tag{3}$$

$$q_2 = -2\pi(x + dx) b \frac{\beta}{360} \lambda \frac{d}{dx}(t + dt). \tag{4}$$

After substituting the values of heat transfer flows (3) and (4) in expression (2) and making the required transformations, we obtain

$$dq = \frac{\lambda\pi\beta b}{180}\left(x\left(\frac{d}{dx}(t+dt) - \frac{dt}{dx}\right) + dx\frac{d}{dx}(t+dt)\right). \tag{5}$$

According to the Lagrange's theorem

$$\frac{d}{dx}(t+dt) - \frac{dt}{dx} = \frac{d^2t}{dx^2}dx. \tag{6}$$

In view of (6), the expression (5) takes the following form:

$$dq = \frac{\lambda\pi\beta b}{180}\left(x\frac{d^2 t}{dx^2} + \frac{dt}{dx} \right)dx \,. \tag{7}$$

On the other hand, the amount of heat removed by convective thermal exchange from the elementary segment will be determined in accordance with the Newton–Richman law:

$$dq = 2\alpha\left(\frac{x\pi\beta}{180} + b \right)(t - t_{\text{В}})dx \,, \tag{8}$$

where α – is the heat transfer coefficient, W/(m^2·°C);
t – is the temperature of the elemental segment, °C;
t_B – is the ambient temperature, which is accepted as t_B = 0 °C.

After substituting the amount of heat removed by convective thermal exchange in accordance with the formula (8) into the expression (7) and performing the required transformations, we obtain

$$\frac{d^2 t}{dx^2} + \frac{1}{x}\frac{dt}{dx} - \frac{2\alpha}{\lambda b}t\left(1 + \frac{180b}{x\pi\beta} \right) = 0 \,. \tag{9}$$

Let us introduce the following notation:

$$\frac{2\alpha}{\lambda b} = m^2 \,;\; mx = z; \; \frac{180b}{\pi\beta}m = \nu \,.$$

Then

$$\frac{1}{x} = \frac{m}{z} \,;\; \frac{dt}{dx} = m\frac{dt}{dz} \,;\; \frac{d^2 t}{dx^2} = m^2\frac{d^2 t}{dz^2} \,. \tag{10}$$

Differential equation (9) in view of (10) will take the following form

$$\frac{d^2 t}{dz^2} + \frac{1}{z}\frac{dt}{dz} - \left(1 + \frac{\nu}{z} \right)t = 0 \,. \tag{11}$$

Let us write down the solution of the differential equation (11) with the use of Mathematica v. 4.2 software suite:

$$t = C_1 e^{-z} U\big(0{,}5(1+\nu);\, 1;\, 2z\big) + C_2 e^{-z} L\big(0{,}5(-1-\nu);\, 2z\big), \tag{12}$$

where C_1 *and* C_2 – are the constant coefficients determined from boundary conditions;
U(0,5(1+v);1; 2z) – is a confluent (degenerate) hypergeometric function of the first kind, of order (1 + v), argument 2z;
L(0,5(–1–v); 2z) – is a Laguerre function of the order of 0.5(–1–v), argument 2z.

Let us determine the constant coefficients C_1 and C_2 in expression (12) from the boundary conditions of the first kind.

For a real saw, let us assume that the temperature will be maximum at a distance $x_0 = \frac{\rho}{\sin \beta/2}$ from the origin (see Fig. 1) and will be equal to t_0.

Then, when $x = x_0$ and $t = t_0$, expression (12) takes the following form

$$t_0 = C_1 e^{mx_0} U\big(0{,}5(1+\nu);1;2mx_0\big) + C_2 e^{mx_0} L\big(0{,}5(-1-\nu);2mx_0\big), \tag{13}$$

and when $x = \infty$ and $t = 0$:

$$0 = C_1 e^{-\infty} U\big(0{,}5(1+\nu);1;\infty\big) + C_2 e^{-\infty} L\big(0{,}5(-1-\nu);\infty\big). \tag{14}$$

Since when $x = \infty$ the Laguerre function $L(0{,}5(-1-\nu);\infty) = \infty$, the expression (14) is valid only when $C_2 = 0$. Then it follows from (13) that

$$C_1 = \frac{t_0}{e^{mx_0} U\big(0{,}5(1+\nu);1;2mx_0\big)}.$$

Given the case that the coefficients C_1 and C_2, as well as x_0 are known, the law of temperature distribution along the height of circular saw tooth can be formulated as follows

$$t = t_0 \frac{e^{mx} U\big(0{,}5(1+\nu);1;2mx\big)}{e^{\frac{m\rho}{\sin \beta/2}} U\left(0{,}5(1+\nu);1;\frac{2m\rho}{\sin \beta/2}\right)}. \tag{15}$$

The graph of the confluent (degenerate) hypergeometric function of the first kind is shown in Fig. 2, the graph of temperature distribution (curve 1) along the height of circular saw tooth is shown in Fig. 3 As can be seen from the formula (15), the nature of temperature distribution does not depend on its absolute value on the tooth blade; therefore, in Fig. 3, the vertical axis shows the relative temperature.

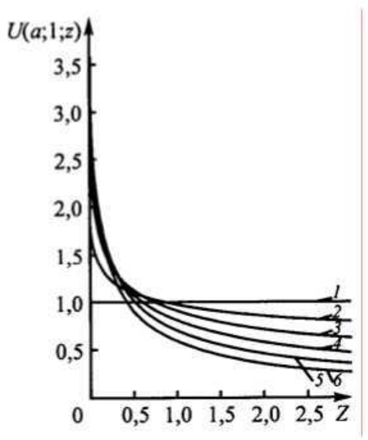
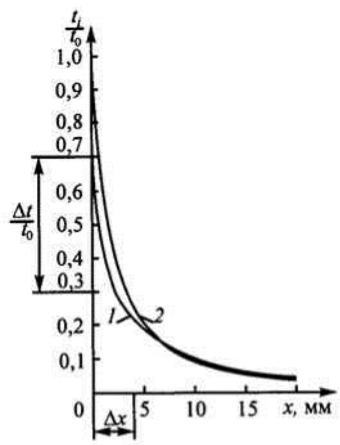


Figure 2. Change of the confluent (degenerate) hypergeometric function of the first kind of argument z, order a: 1 – a = 0; 2 – 0.2; 3 – 0.4; 4 – 0.6; 5 – 0.8; 6 – 1.0



мм	mm

Figure 3. Distribution of relative temperature along the height of circular saw tooth: 1 – curve plotted according to the formula (15); 2 – curve plotted with the use of grapho-analytical method [3]

Conclusions

1. The nature of the change in the relative temperature along the height of circular saw tooth obtained by analytical calculation (curve 1) is consistent with the results of solving the inverse problem for the same cutting conditions with the use of grapho-analytical method (curve 2). This allows us to conclude that the real picture of the stationary temperature field of the tooth of the cutting part of circular saw has been established.

2. An analytical method for the calculation of tooth temperatures is recommended for the cases when the cutting temperature is known, the grapho-analytical method is recommended for the cases when the disk temperature at the dedendum circle is known.

3. Temperature gradients along the height of circular saw tooth (curves *1, 2)* are most distinct in coordinate segments along the x-axis $\Delta x = 0–4$ mm; $\Delta t/t_0 = 0.3–0.7$. This can be explained by the assumptions made in the description of the thermophysical situation in the cutting zone for the development of an analytical method, which does not take into account the closed cutting process, the presence of warm chips, heat transfer by irradiation. It should be noted that the temperature gradient in the idling section of the saw is higher than in the working section.

4. The error in determining the temperature of the base of the tooth by the analytical method, being adopted in the calculation of the cutting temperature does not exceed 6%. It can be concluded that the real temperature at the collar of the disk is sufficiently accurate and can be used for the calculation of cutting conditions based on the dynamic stability of the saw.

5. The proposed method for the calculation of temperatures of circular saw tooth makes it possible to scientifically recommend the limitations of cutting parameters for circular saws taking into account the material of the tooth blade and the tempering technique used during its manufacture.

REFERENCES

1. *Sh. Sh. Abelsky* Temperature field of rotating flexible disks [Text] / Sh. Sh. Abelsky, V. K. Pashkov // Inzhenerno-Fizichesky Zhurnal. – Vol. XX, No. 2. – pp. 338–343.

2. *V. N. Konov* Study of the effect of thermal phenomena on the performance of circular saws [Text]: extended abstract of dissertation of Candidate of Technical Sciences 13/00/01 / Viktor Nikolayevich Konov; Saint Petersburg State Forestry University. – Leningrad State Publishing House, 1979. – 26 p.

3. *V. K. Pashkov* Analytical method for the calculation of temperature field of circular saw tooth [Text] / V. K. Pashkov, S. V. Shchepochkin // Socio-economic and environmental problems of the forest complex. – Yekaterinburg: Ural State Forest Engineering University, 2003. – pp. 128–130.

4. *A. N. Reznikov* Thermal physics of wood cutting [Text] / A. N. Reznikov – Moscow: Mashinostroyenie Publishing House, 1969. – 288 p.

5. *Hanptmann, E.* Temperature Distribution in a Thin Rotating Disk [Text] / E. Hanptmann, H. Ramsey // Appl. Sii. Res., 1969. – 120 p.

6. *Kalitzin, G.* Erularmung und Warmes pannungen des Krejssage blattes [Text] / G. Kalitzin // Holzindustrie. – 1956. – N 11. – P. 269–292.

Ural State Forest Engineering University

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V. K. Pashkov, S. V. Shchepochkin

Ural State Forest-Technical University

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